

INTRO TO DERIVATIVES: OPTIONS

BRAINTEASER!

There are 8 people sitting in a room at an investment bank. 4 of them are traders and 4 of them are bankers. They are split off into 4 teams of 2 for team-building exercises. What is the probability that each group of 2 has 1 trader and 1 banker?

ANSWER: PROBABILITY APPROACH

Select any person to be the first member of the group. 8/8 ways to do this. For their partner there are 4/7 valid people to select. Repeat for 2nd group, with 6/6 ways to select the first person and 3/5 ways to select the 2nd person. Same logic applies for 3rd group, for a total of $4/7 * 3/5 * 2/3 = 24/105 = \mathbf{8/35}$. Note that we don't have to worry about the 4th group because there will naturally be 1 banker and 1 trader left if the other groups are valid.

ANSWER: COMBINATIONS APPROACH

The total number of combinations is $8 \text{ choose } 2 * 6 \text{ choose } 2 * 4 \text{ choose } 2$ divided by $4!$ because the selection of groups does not require order. This comes out to 105 combinations. The number of valid groups can be thought of as $4*4$ for group 1, $3*3$ for group 2, $2*2$ for group 3. Again divided by $4!$ because the order does not matter. This comes out to 24 valid combinations, for a total probability of $24/105 = \mathbf{8/35}$

WHAT IS A DERIVATIVE?

Derivative

- A derivative is a financial instrument whose value is based on the value of another underlying asset
- When the price of the underlying changes, the value of the derivative also changes

WHAT IS A DERIVATIVE?

Types of Derivatives

- Forwards/Futures
- Options
- Swaps
- Warrants/Convertibles

CALL OPTION

- An agreement that gives the buyer the right, but not the obligation, to buy an underlying asset at a specified price within a specific time period

PUT OPTION

- An agreement that gives the buyer the right, but not the obligation, to sell an underlying asset at a specified price within a specific time period

Let's say you believe that the stock of Company XYZ, currently trading at \$50, is going to rise in the next three months. You decide to purchase a call option on XYZ stock that expires in three months with a strike price of \$55 for a premium (the price of the option) of \$3.

- **Scenario 1: Stock Price Rises Above the Strike Price**

- If, after two months, XYZ stock rises to \$65, your call option allows you to buy the stock at \$55, even though it's now worth \$65 on the open market. You could exercise the option, purchasing the stock at \$55, and then sell it immediately in the market for \$65, securing a profit.
- Your profit would be the selling price minus the strike price minus the premium paid:
$$\$65 - \$55 - \$3 = \$7 \text{ per share.}$$

- **Scenario 2: Stock Price Remains Below the Strike Price**

- If, at expiration, XYZ stock is trading at \$53, the call option is "out of the money," because the stock is trading below the strike price. Exercising the option would mean buying the stock for more than it's currently worth, so you choose not to exercise the option. Your loss is limited to the premium you paid for the option, which is \$3 per share.

OTHER KEY TERMS TO KNOW

- S** - Price of Underlying Asset
- F** - Forward (Futures) Price of Underlying Asset
- K** - Strike (Exercise) Price
- t** - Time to Expiration
- r** - Rate of Interest
- σ** - Volatility
- C** - Call Price
- P** - Put Price
- q** - Dividend Yield

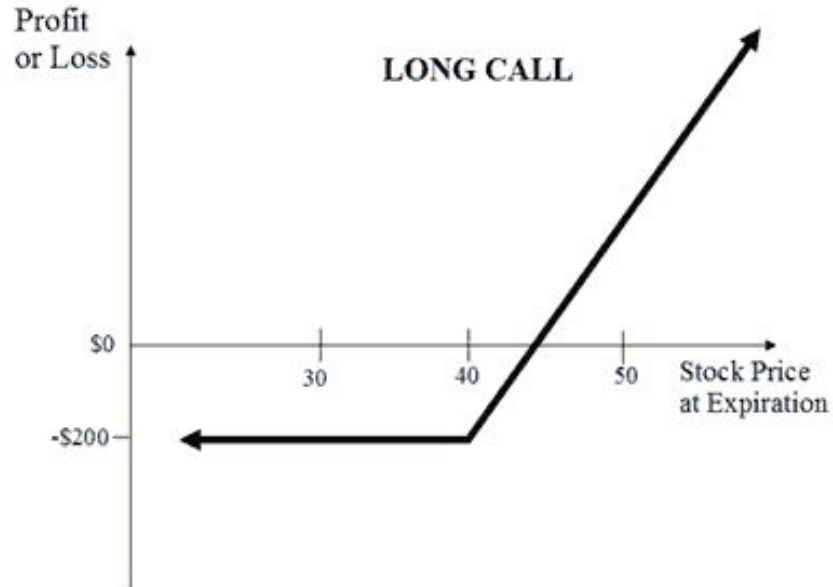
What is the difference between the price of the underlying asset and the strike price?

2 TYPES OF OPTIONS

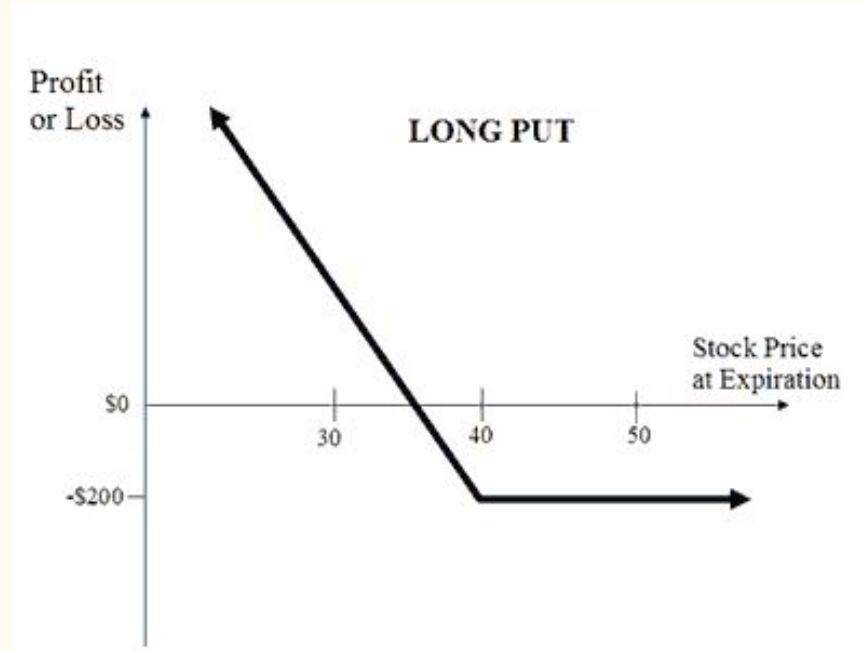
American Options – buyer can exercise the option early, at any point up until expiration

European Options – buyer cannot exercise option early and has to wait until expiration

PAYOFF DIAGRAM FOR BUYING A CALL



PAYOFF DIAGRAM FOR BUYING A PUT



CONCEPT OF MONEYNES

- Options are heavily dependent on the concept of moneyness – relative position of the price of the underlying asset with respect to the strike price of the option

CONCEPT OF MONEYNES

- In the Money
 - If expiration were today, the option would have value
- At the Money
 - Strike price = Spot price
- Out of the Money
 - If expiration were today, the option would be worthless

WHY PEOPLE TRADE OPTIONS

- Leverage
- Hedging – Protected downside risk
- Speculation

Options Chain Table

Calls			strike	Puts		
bid	theo	ask		bid	theo	ask
7.75	8.00	8.25	315	5.75	6.00	6.25
5.25	5.50	5.75	320	8.25	8.50	8.75

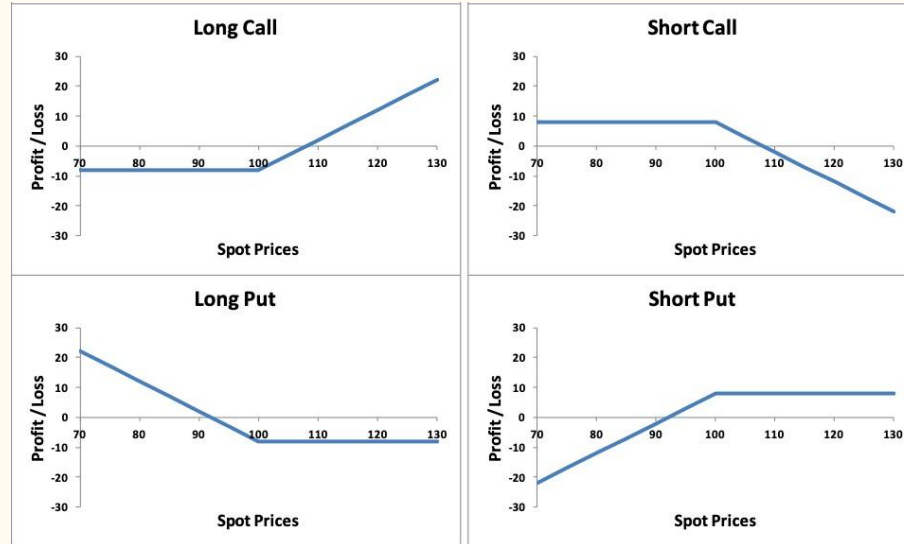
55.11	+0.62	+1.14%	14	2,248	265.00	18.30	-1.42	-7.20%	4	4,110
52.45	+0.45	+0.87%	65	4,996	270.00	20.00	-0.85	-4.08%	30	8,071
49.20	+0.65	+1.34%	47	1,738	275.00	22.00	-1.30	-5.58%	4	3,448
45.55	+0.10	+0.22%	56	13,643	280.00	23.90	-0.90	-3.63%	77	11,761
42.45	+0.45	+1.07%	18	1,190	285.00	25.35	-1.13	-4.27%	19	5,260
39.55	+0.38	+0.97%	168	7,599	290.00	27.75	-1.60	-5.45%	44	10,112
36.65	+0.41	+1.13%	18	1,291	295.00	29.85	-1.45	-4.63%	35	871
33.99	+0.63	+1.89%	1,164	17,550	300.00	32.30	-1.57	-4.64%	82	8,980
29.00	+0.40	+1.40%	263	9,489	310.00	37.01	-1.69	-4.37%	12	2,583
24.20	+0.10	+0.41%	426	14,999	320.00	42.45	-1.54	-3.50%	6	2,464
20.60	+0.30	+1.48%	666	10,833	330.00	48.33	-1.49	-2.99%	3	2,182
16.65	-0.15	-0.89%	281	6,361	340.00	54.98	-1.52	-2.69%	7	1,096
13.80	+0.12	+0.88%	914	11,849	350.00	61.62	-3.08	-4.76%	35	670
11.25	-0.10	-0.88%	39	6,537	360.00	69.61	-0.84	-1.19%	6	236
9.10	-0.16	-1.73%	130	3,973	370.00	76.48	-10.18	-11.75%	3	338
7.10	-0.25	-3.40%	41	5,192	380.00	84.62	-10.26	-10.81%	3	224

Should the right to buy for 300 and the right to sell at 300 be worth about the same, given the current stock price of 303.49?

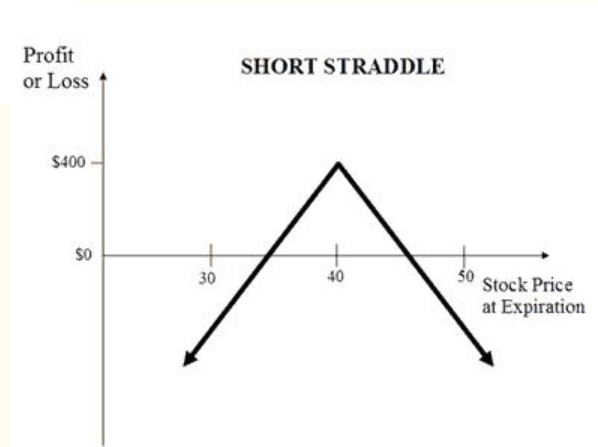
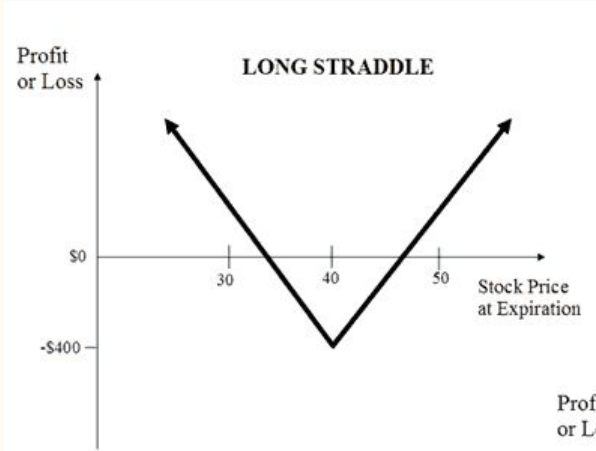
FUTURE TOPICS

- Futures/Forwards
- Options on Futures
- Greeks

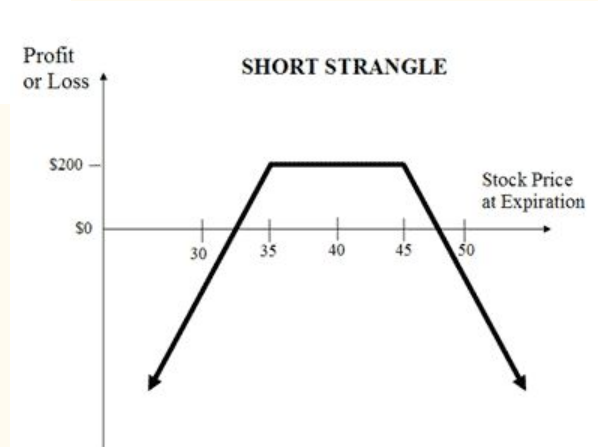
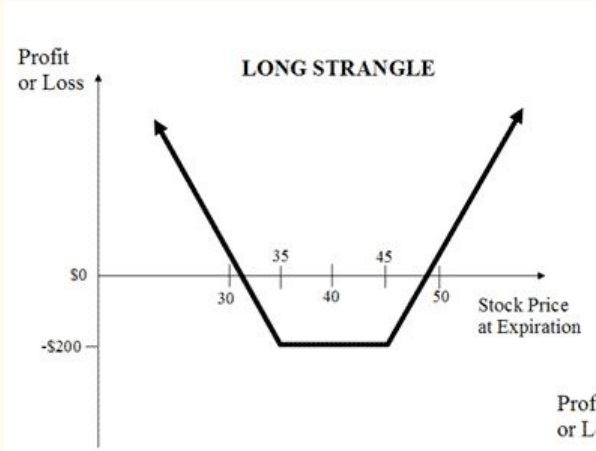
SUMMARY OF PAYOFF DIAGRAMS



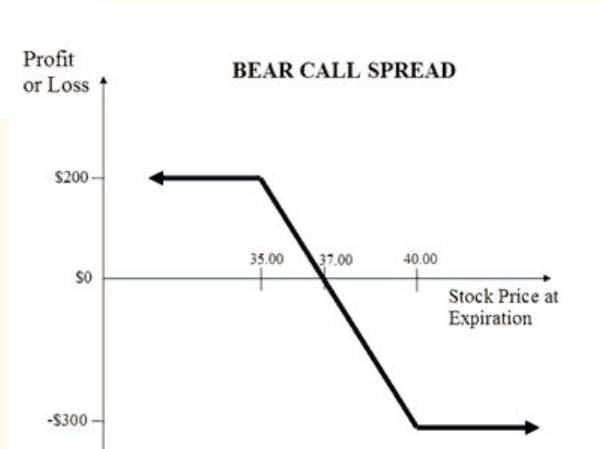
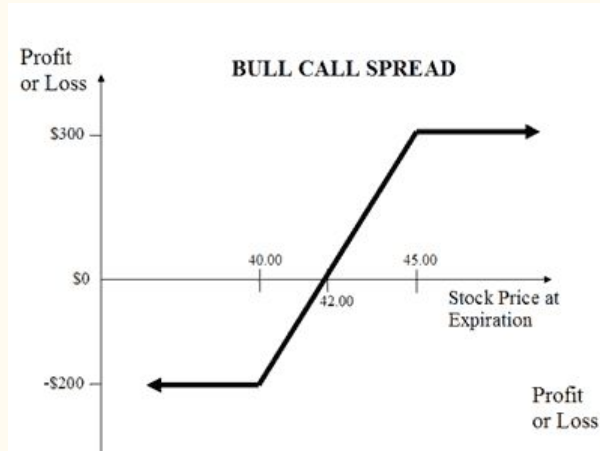
DIFFERENT STRATEGIES: VOLATILITY STRATEGIES



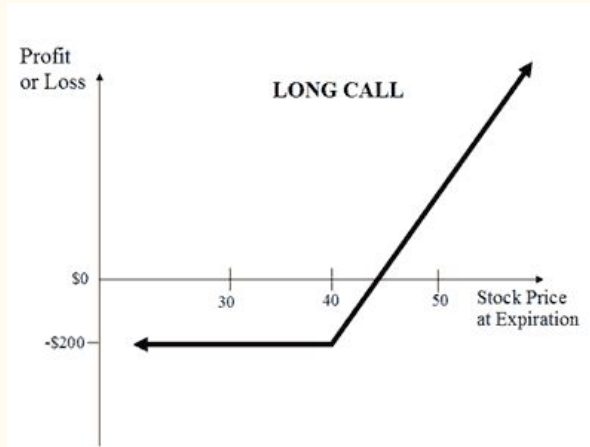
DIFFERENT STRATEGIES: VOLATILITY STRATEGIES



DIFFERENT STRATEGIES:



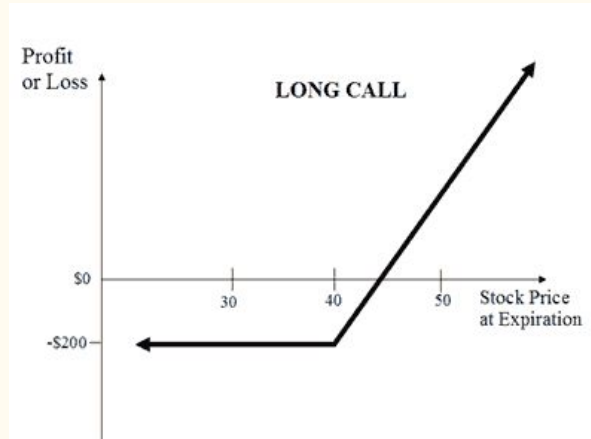
PRICING EXERCISE



How much do you think a call option should be priced if...

- Strike price is 40
- Price of underlying is 50
- Expiration date is in a year

PRICING EXERCISE



Black-Scholes Value:

10.932

Stock Price:
(in USD)

(ex. 31.55)

Exercise Price:
(in USD)

(ex. 22.75)

Time to maturity:
(in years)

(ex. 3.5)

Annual risk-free interest rate

(ex. 5%)

Annualized volatility

(ex. 50%)

Black-Scholes Value:

14.963

Stock Price:
(in USD)

(ex. 31.55)

Exercise Price:
(in USD)

(ex. 22.75)

Time to maturity:
(in years)

(ex. 3.5)

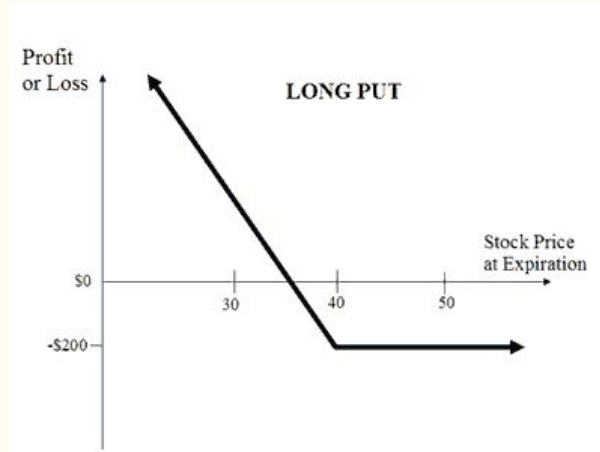
Annual risk-free interest rate

(ex. 5%)

Annualized volatility

(ex. 50%)

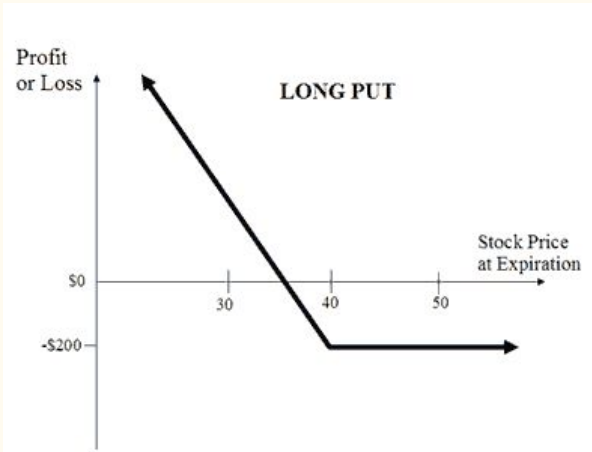
PRICING EXERCISE



How much do you think a put option should be priced if...

- Strike price is 40
- Price of underlying is 40
- Expiration date is in a year

PRICING EXERCISE



Spot Price (SP)	<input type="text" value="40"/>
Strike Price (ST)	<input type="text" value="40"/>
Time to Expiration (t)	<input type="text" value="1"/> <input type="text" value="Years"/>
Volatility (v)	<input type="text" value="20"/> %
Risk-Free Interest Rate (r)	<input type="text" value="1"/> %
Dividend Yield (d)	<input type="text" value="0"/> %

Put Price: **\$2.98**

Spot Price (SP)	<input type="text" value="40"/>
Strike Price (ST)	<input type="text" value="40"/>
Time to Expiration (t)	<input type="text" value="1"/> <input type="text" value="Years"/>
Volatility (v)	<input type="text" value="50"/> %
Risk-Free Interest Rate (r)	<input type="text" value="1"/> %
Dividend Yield (d)	<input type="text" value="0"/> %

Put Price: **\$7.66**

PRICING RELATIONSHIPS

- $\text{Max}[0, S-K] \leq C$
- $C(K_L) > C(K_H)$
- $C(t_2) > C(t_1)$
- $\text{Max}[0, K-S] \leq P \leq K$
- $P(K_L) < P(K_H)$
- $P(t_2) > P(t_1)$
- $S, K = \text{spot price, strike price}$
- $K_L, K_H = \text{lower strike, higher strike}$
- $T_1, T_2 = \text{shorter maturity, longer maturity}$

IMPACT OF EACH VARIABLE

- What happens to the cost of a call option when the following variables change?
 - Price of underlying increases...
 - Strike price increases...
 - Volatility increases...
 - Time increases...
 - Interest rates increase...
 - Dividends increase...

IMPACT OF EACH VARIABLE

- What happens to the cost of a put option when the following variables change?
 - Price of underlying increases...
 - Strike price increases...
 - Volatility increases...
 - Time increases...
 - Interest rates increase...
 - Dividends increase...

PUT-CALL PARITY

General approach: prices do not provide arbitrage opportunities

Derivation obtained by replicating the payoffs provided by the option using the underlying asset and borrowing/lending. The option payoffs should be priced the same as the replicated payoffs.

The same approach as in pricing futures/forward

PUT-CALL PARITY

- Defines the relationship between the price of a European put and European call of the same class (same strike/underlying asset/expiration date)

